

## Envelope evolution of a laser pulse in an active medium

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We show that the envelope velocity,  $v_{\text{env}}$ , of a short laser pulse can, via propagation in an active medium, be made less than, equal to, or even greater than  $c$ , the vacuum phase velocity of light. Simulation results, based on moving frame propagation equations coupling the laser pulse, active medium, and plasma, are presented, as well as equations that determine the design of superluminal and subluminal  $v_{\text{env}}$  values. In this simulation the laser pulse evolves in time in a moving frame as opposed to earlier work [D. L. Fisher and T. Tajima, *Phys. Rev. Lett.* **71**, 4338 (1993)], where the profile was fixed. The elimination of phase slippage and pump depletion effects in a laser wake field accelerator is discussed as a particular application. Finally, we discuss media properties necessary for an experimental realization of this technique.

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### I. INTRODUCTION

It has been suggested [1] that an active laser medium may be used to control the envelope propagation velocity ( $v_{\text{env}}$ ) of a laser pulse in an active (active-plasma) medium to any design velocity, including the speed of light or greater. Such a pulse is said to be superluminal. It is possible with appropriate construction of the laser pulse profile in the direction of propagation to "accelerate" the group velocity of that pulse [2,3]. This is related to the process of self-induced transparency [4] and to the triple soliton solutions already obtained for a (nonactive) plasma system [5,6]. The preexcited active medium plays the role of a nonlinear amplifier, amplifying the front of the laser pulse and absorbing energy at the rear of the pulse in such a manner as to maintain the pulse shape but at the same time to increase the overall pulse speed. This apparent acceleration of the group velocity even beyond the speed of light does not violate the special theory of relativity, as energy and information flow in fact does not exceed  $c$ . The leading edge of the pulse, which is necessary for acceleration, already contains information about the pulse, and this information is extracted through the nonlinear amplification process. The energy to form the envelope comes from two sources: one is the normal energy flow of the existing laser pulse that flows at the normal group velocity ( $v_g < c$ ), and the other contribution for the envelope acceleration is the energy already present in the active medium. The normal group velocity is the laser pulse velocity in the absence of an active medium.

The process of effective acceleration of the photon group velocity and the recovery of laser energy loss by the active medium has applications in photonics and telecommunications, as well as to wake-field accelerators [7]. Concerning applications to wake-field acceleration, since the pulse can travel at the design speed with no change in its structure, the phase velocity of the accelerating field will also be at the design speed in an active-plasma medium. The increased phase velocity of

the wake field could help overcome slippage between the particle bunch and accelerating field, as the particle bunch otherwise outruns the accelerating field due to the difference between the phase velocity of the field and the particle velocity  $v \approx c$ . Second, since the energy used to induce the wake comes from the active medium and not from the laser pulse, pump depletion [8] would be alleviated. Third, using a properly shaped active medium channel in the transverse direction, we should be able to replenish energy loss due to diffraction, thereby optically guiding and effectively focusing the laser pulse without relying on self-focusing [9,10] of the laser pulse.

In the previous paper [1] we considered only stationary structure and not its evolution and stability. Here we consider both stationary and nonstationary structures and their stability and evolution. This paper looks at some of the questions regarding more realistic conditions and parameters for the laser pulse. First the relationship between the energy density of the medium and design velocity is considered in Sec. II. Then effects of evolving from a vacuum into the medium and the effect of finite length are investigated in Sec. III. Applications to wake-field acceleration and effects of a Gaussian laser profile are also investigated in Sec. IV. In Sec. V we discuss possible parameters for and preparation of an active-plasma medium.

### II. THEORY

We start with the electromagnetic wave equation in the Coulomb gauge,

$$\left[ \nabla^2 - \frac{\eta^2}{c^2} \frac{\partial^2}{\partial t^2} \right] \vec{A} + c \vec{\nabla} \frac{\partial \phi}{\partial t} = -\frac{4\pi \vec{j}}{c} - \frac{4\pi}{c} \frac{\partial \vec{P}}{\partial t}, \quad (1)$$

where  $\vec{A}$  is the vector potential of the laser pulse,  $\phi$  is the electrostatic potential for the plasma electric field, and the transverse component of polarization vector  $\vec{P}$  contains only the active material dependent (atomic) response. None of the plasma effects are included in  $\vec{P}$ ; they enter through  $\phi$  and  $J$ . The index of refraction  $\eta$  is

for the nonactive portion of the material response (atomic). The form of the fields in one dimension for a laser of frequency  $\omega_0$  and wave number  $k_0$  are

$$P = [U(z, t) + iV(z, t)]e^{i[k_0 z - \omega_0 t + \psi(z, t)]}, \quad (2)$$

$$A = a(z, t)ie^{i[k_0 z - \omega_0 t + \psi(z, t)]}, \quad (3)$$

$$J = -\frac{e^2 n A}{\gamma m c}, \quad (4)$$

$$\gamma = \frac{1}{\left[1 - \frac{\vec{v} \cdot \vec{v}}{c^2}\right]^{1/2}}, \quad (5)$$

where  $\vec{v}$  is the velocity of the plasma,  $U(z, t)$  and  $V(z, t)$  are the slowly varying real (electric dipole dispersion) and imaginary (absorption) components of the material polarization vector [4], and  $\psi(z, t)$  and  $a(z, t)$  are the slowly varying phase and envelope of the laser pulse. The trans-

verse current  $J$  is obtained from the conservation of canonical momentum and the fact that initially the plasma is stationary. With the assumption that the laser oscillation time and length scales are much shorter than those for  $a(z, t)$ ,  $\psi(z, t)$  and the plasma ( $\omega_p$ ), we multiply Eq. (1) by  $\vec{A}^*$  and average over the fast laser scales [4].

In our previous paper [1] we were interested in the exact stationary shape and transformed to a moving frame  $\lambda = z - ct$  without looking at the evolution of the pulse. However, since we are now interested in the evolution of a laser pulse traveling at approximately  $v_g \approx c$ , we allow explicit time evolution of the envelope of the pulse and transform to the moving coordinates  $\lambda = z - ct$  and  $t = t$ . For simplicity we look at only the one dimensional (1D) regime. In this moving frame the envelope is slowly varying and we assume  $\partial^2/\partial t^2 \ll \omega_0(\partial/\partial t)$ . Dropping the  $\partial^2/\partial t^2$  terms and using the zeroth order dispersion relation  $\omega_0^2 = k_0^2 c^2 + \omega_p^2/\gamma$ , the transformation gives the evolution equations for the slowly varying quantities  $a$  and  $\psi$ ,

$$2a \frac{\eta^2}{c^2} \frac{\partial \psi}{\partial t} \left[ \omega_0 + c \frac{\partial \psi}{\partial \lambda} \right] = (1 - \eta^2) \frac{\partial^2 a}{\partial \lambda^2} - 2a \left[ k_0 - \frac{\eta^2 \omega_0}{c} \right] \frac{\partial \psi}{\partial \lambda} - a(1 - \eta^2) \left[ \frac{\partial \psi}{\partial \lambda} \right]^2 + \frac{2\eta^2}{c} \frac{\partial^2 a}{\partial \lambda \partial t} + \frac{\eta^2 a}{c^2} \left[ \frac{\partial \psi}{\partial t} \right]^2 - \frac{4\pi}{c} \frac{e^2 a \delta n}{m \sqrt{1 + (ea^2/mc^2)}} + \frac{4\pi}{c} \left[ \frac{\partial V}{\partial t} - c \frac{\partial V}{\partial \lambda} \right], \quad (6)$$

$$\frac{2\eta^2}{c^2} \left[ \omega_0 + c \frac{\partial \psi}{\partial \lambda} - \frac{\partial \psi}{\partial t} \right] \frac{\partial a}{\partial t} = 2 \left[ \frac{\omega_0 \eta^2}{c} - k_0 \right] \frac{\partial a}{\partial \lambda} - \frac{2\eta^2}{c} \frac{\partial a}{\partial \lambda} \frac{\partial \psi}{\partial t} - \frac{2\eta^2 a}{c} \frac{\partial^2 \psi}{\partial \lambda \partial t} + \frac{4\pi}{c} \left[ V \left[ \omega_0 + c \frac{\partial \psi}{\partial \lambda} - \frac{\partial \psi}{\partial t} \right] \right] - (1 - \eta^2) a \frac{\partial^2 \psi}{\partial \lambda^2}, \quad (7)$$

where  $\delta n$  is the electron density variation from the uniform equilibrium density  $n_0$ .

We consider the laser medium in an approximate fashion, taking into account only two levels of interest. This approximation is valid when the interacting electromagnetic wave frequency is at or near resonance and the natural widths of the levels are sufficiently narrow. Laser medium equations for a two state system with dipole transitions at exact resonance ( $\mathcal{E}_{\text{res}} = \hbar\omega_0$ ) without relaxation may be given by [4]

$$\frac{\partial W}{\partial t} = c \frac{\partial W}{\partial \lambda} + \frac{\kappa^2}{c} a W, \quad (8)$$

$$\frac{\partial W}{\partial t} = c \frac{\partial W}{\partial \lambda} - \frac{\omega_0^2}{c} a V, \quad (9)$$

where  $\mathcal{E}_{\text{res}}$  is the atomic transition energy between the two states under consideration,  $\hbar\kappa$  is the dipole moment for the active state, and the time constants for incoherent damping effects and energy damping of the active system are assumed to be long compared to the interaction time. The energy density  $W$  of the active medium is defined with respect to the ground state energy density  $W_0 = -N_a(\hbar\omega_0/2)$ , where  $N_a$  is the density of possible active states.

The longitudinal laser-plasma interaction is modeled through the ponderomotive force in the electron momentum equation (11), and the ions are assumed immobile. The continuity equation takes the form

$$\frac{\partial n}{\partial t} = c \frac{\partial [n(1 - v/c)]}{\partial \lambda}, \quad (10)$$

and the momentum equation is

$$\frac{\partial \gamma v}{\partial t} = c \left[ 1 - \frac{v}{c} \right] \frac{\partial \gamma v}{\partial \lambda} - \frac{eE}{m} + \frac{e^2}{m^2} \frac{1}{2\gamma} \frac{\partial a^2}{\partial \lambda}. \quad (11)$$

Poisson's equation becomes

$$\frac{\partial E_p}{\partial \lambda} = 4\pi e n_0 \left[ \frac{n}{n_0} - 1 \right], \quad (12)$$

where  $E_p$  is the longitudinal electric field of the plasma driven by the ponderomotive force of the laser. Equations (10)–(12) involve the longitudinal motion of the plasma, while the transverse motion of the plasma directly related to the laser electromagnetic fields is already incorporated in Eqs. (6)–(9) in the current 1D model. The last seven equations are numerically integrated with specific initial conditions, and results are given in Sec. III.

Three conditions must be met to accelerate the pulse envelope to the design speed  $v_{\text{env}}$  and retain a stationary structure. The first concerns the resonance condition: the laser photon energy should approximately match that of the transition energy in the active medium. Here we consider the case of exact resonance, but small detuning might be both possible and desirable to minimize unwanted nonlinear optical effects. Second, the laser pulse duration must match the time scale of energy exchange between the active medium and laser pulse (reciprocal of the average Rabi frequency). Third, a certain energy density in the active medium is necessary for the acceleration of the pulse envelope.

The third condition states that there must be enough energy  $\Gamma$  stored in the active medium prior to the pulse arrival to amplify the leading edge of the pulse to the peak value in the distance  $L_v$  [ $L_v = l_{\text{pulse}} v_{\text{env}} / (v_{\text{env}} - v_g)$ ] for the accelerated pulse envelope to outrun the otherwise nonaccelerated pulse over the pulse length  $l_{\text{pulse}}$ . That is, there must be energy equal to that of the pulse stored in the length  $L_v$  of the active medium. With proper conditioning of the energy density of the active medium, the envelope velocity of the laser pulse may be either accelerated or decelerated from the laser pulses normal group velocity. When the atomic effect dominates  $1/\eta < \sqrt{1 - (\omega_p/\omega_0)^2}$  with  $\eta \approx 1$  and  $\omega_0/\omega_p \gg 1$  (underdense plasma), this yields

$$W(-\infty) + |W_0| = \Gamma_v \approx \frac{l_{\text{pulse}} E^2}{L_v 4\pi}. \quad (13)$$

Therefore from the definition of  $L_v$  we have

$$W(-\infty) + |W_0| = \left[ 1 - \frac{c}{\eta v_{\text{env}}} \right] \frac{E^2}{4\pi}, \quad (14)$$

where  $W(-\infty)$  is the value of the energy density  $W$  prior to the arrival of the pulse,  $\Gamma_v$  is the energy density necessary to accelerate the stationary structure to the design velocity  $v_{\text{env}}$ , and  $E$  is the transverse electric field of the laser. The interaction between the laser pulse and active medium is characterized by two time scales, the average inverse Rabi frequency [11]  $\Omega_R^{-1}$  and the laser pulse scale  $\Omega_l^{-1} = l_{\text{pulse}}/2\pi c$ . The frequency of energy exchange between the laser and active medium ( $\Omega_R$ ) is derived using Eqs. (8) and (9) along the approximate average laser intensity  $a^2/2$  stationary in the moving frame to yield

$$\frac{\partial^2 V}{\partial \lambda^2} = -\frac{\kappa^2 \omega_0^2 a^2}{2c^4} V. \quad (15)$$

This gives  $\Omega_R \approx \kappa \omega_0 a / \sqrt{2} c^2$ . Additionally the laser time scale comes from Eq. (7); keeping only leading order terms yields

$$\frac{\partial^2 a}{\partial \lambda^2} = \frac{4\pi \kappa^2 W a}{(\eta^2 - 1)c^2}. \quad (16)$$

Using Eq. (14) ( $v_{\text{env}} = c$ ) for the energy density gives  $\Omega_l \approx \kappa \omega_0 a / \sqrt{2} c^2$ . The second necessary condition

$$\Omega_R = \Omega_l \approx \frac{\kappa \omega_0 a}{\sqrt{2} c^2} \quad (17)$$

for a stationary superluminal structure is in fact demonstrated: these two time scales must match. This match is necessary since the energy must return to the active medium on the average Rabi time scale. If the Rabi time scale does not match the duration of the laser pulse, the energy return will be incomplete if the pulse is short and will again extract energy if the pulse is too long.

When the plasma effect dominates over the atomic effect  $1/\eta \gg [1 - (\omega_p/\omega_0)^2]$ , the stored energy for acceleration to the design velocity becomes

$$\begin{aligned} W(-\infty) + |W_0| = \Gamma_v &\approx \frac{l_{\text{pulse}} E^2}{L_v 4\pi} \\ &\approx \left[ 1 - \frac{c}{v_{\text{env}}} \left[ 1 - \frac{\omega_p^2}{2\omega_0^2} \right] \right] \frac{E^2}{4\pi}, \end{aligned} \quad (18)$$

since the group velocity of a laser pulse in a plasma is  $v_g = c \sqrt{1 - \omega_p^2/\omega_0^2} \approx c(1 - \omega_p^2/2\omega_0^2)$ . Setting the velocity of the laser pulse envelope  $v_{\text{env}} = c$  in Eq. (18) yields

$$\Gamma_c = W(-\infty) + |W_0| \approx \frac{\omega_p^2 E^2}{2\omega_0^2 4\pi}. \quad (19)$$

For a stationary laser pulse structure, the energy density of the active medium  $W(-\infty)$  must replenish the energy lost by the laser pulse due to the production of the plasma wake field (pump depletion). That is, to be stationary, the pulse must lose no energy. For optimal wake field production [7] from a laser pulse we assume  $l_{\text{pulse}} \approx \lambda_p$ , where  $\lambda_p = 2\pi c/\omega_p$ . The energy density of the active medium necessary to replenish and overcome this laser pump depletion ( $W_{\text{pump}}$ ) is given by

$$W_{\text{pump}} \approx \frac{E_p^2}{4\pi}, \quad (20)$$

where  $E_p$  is given by Eq. (11). Using this with Eq. (19) gives

$$\begin{aligned} W_{\text{pump}}(0) &\approx \left[ \frac{\omega_p}{\omega_0} \right]^2 \left[ \frac{eE}{m\omega_p c} \right]^2 \frac{\omega_p^2 E^2}{2\omega_0^2 4\pi} \\ &= \left[ \frac{\omega_p}{\omega_0} \right]^2 \left[ \frac{eE}{m\omega_p c} \right]^2 \Gamma_c. \end{aligned} \quad (21)$$

In all regimes for acceleration  $(\omega_p/\omega_0)^2 (eE/m\omega_p c)^2 \ll 1$ . Therefore, the energy density necessary for acceleration to  $c$  is much larger than that to overcome pump depletion.

It is foreseeable that very short laser pulses may be produced such that the pulse length is short compared to the ionization time for that material ( $\tau_i \gg \tau_{\text{pulse}}$ ). We note the recent progress of ultrashort laser pulse technology [12,13] in particular. This could allow  $N_a/n_0 \gg 1$  with a very short plasma wavelength and therefore very large accelerating gradients with a phase velocity equal to  $c$ . Also with a properly prepared short pulse it may be possible to induce and absorb the plasma oscillation within the laser pulse, producing no wake and therefore not destroying dense material.

### III. SIMULATION AND RESULTS

Equations (6)–(12) were numerically integrated with initial conditions. The initial conditions included material dependent constants, the dipole moment  $\kappa$ , the initial imaginary part of the polarization  $V_0$ , energy density  $W_0$ ,  $\omega_0/\omega_p$ , and the index of refraction  $\eta$  for the inactive portion of the response of the material. In the simulations all the active states are considered to be initially full or depleted. This implies that the imaginary part of the polarization  $V_0=0$  and the initial energy density  $=\pm W_0$ , where for all states initially active we have  $+W_0$  and for all states initially depleted  $-W_0$ . The initial condition of a partially full energy density corresponds to the initial imaginary part of the polarization not equal to zero ( $V_0\neq 0$ ). To simulate realistic conditions, we have the laser pulse starting outside of the active medium (active-plasma medium) traveling into the medium.

In Ref. [1] we demonstrated that there is a stable structure ( $\approx$ sech). However, to simulate most experiments, it is necessary to consider the evolution of the laser pulse under realistic conditions. The first of these properties is the evolution of the laser pulse from vacuum into the active medium (active-plasma medium). In all the simulations the laser pulse is initialized in a vacuum and evolves into the active (active-plasma) medium, which has a Gaussian edge profile. Since the leading edge of the laser pulse enters the active medium ahead of the trailing edge of the pulse, it is amplified while the tail is not absorbed, increasing its width. Therefore, the initialization of the width of the laser pulse should be to a value less than the predicted value, due to this broadening of the laser pulse as it enters the active medium. The higher the gain of the medium, the larger the initialized reduction in width. In all the simulations this had no noticeable effect on the long term stability of the laser pulse, whether the pulse was a sech or Gaussian. The first parameter we checked is the relationship between the energy density ( $W_0$ ) and the envelope velocity of the laser pulse. In Table I we show the results of different simulation runs with an energy density  $W_0$  given by Eq. (14) for a sech laser pulse with different  $v_{\text{env}}/c$ . For these cases  $\omega_0/\omega_p = 100$  so that the plasma will have little effect on the velocity ( $v_g = 0.999995c$ ). The other parameters were  $a = 0.1mc^2/e$ ,  $\kappa = 0.19e/mc$ ,  $\eta = 1$ . From this table we see that the results compare very well with the theory.

TABLE I. The theoretical initial energy density ( $W_0$ ) for different  $\beta = v_{\text{env}}/c$  from Eq. (14) compared to the simulation  $\beta$  for that initial energy density. Initial conditions were  $\eta = 1$ ,  $a = 0.1mc^2/e$ ,  $\omega_0/\omega_p = 100$ , and  $\kappa = 0.19e/mc$  for a sech laser pulse.

Design $\beta$	$W_0$	Simulation $\beta$
0.980	-1.0204	0.980
0.990	-0.505	0.990
1.000	0.00	1.000
1.010	0.50	1.010
1.020	0.98	1.020
1.030	1.456	1.029

For “deceleration” of the envelope velocity we start with the initial conditions of a negative energy density and the imaginary part of the polarization vector equal to zero. An energy density  $W_0 < 0$  (with  $V_0 = 0$ ) corresponds to an active medium with no states active. In this case as the laser pulse enters an inactive medium, the process of self-induced transparency takes place [4], reducing the envelope velocity to less than  $c$ . In all the cases, only the energy density was varied to produce the acceleration and all other parameters were left unchanged. The width

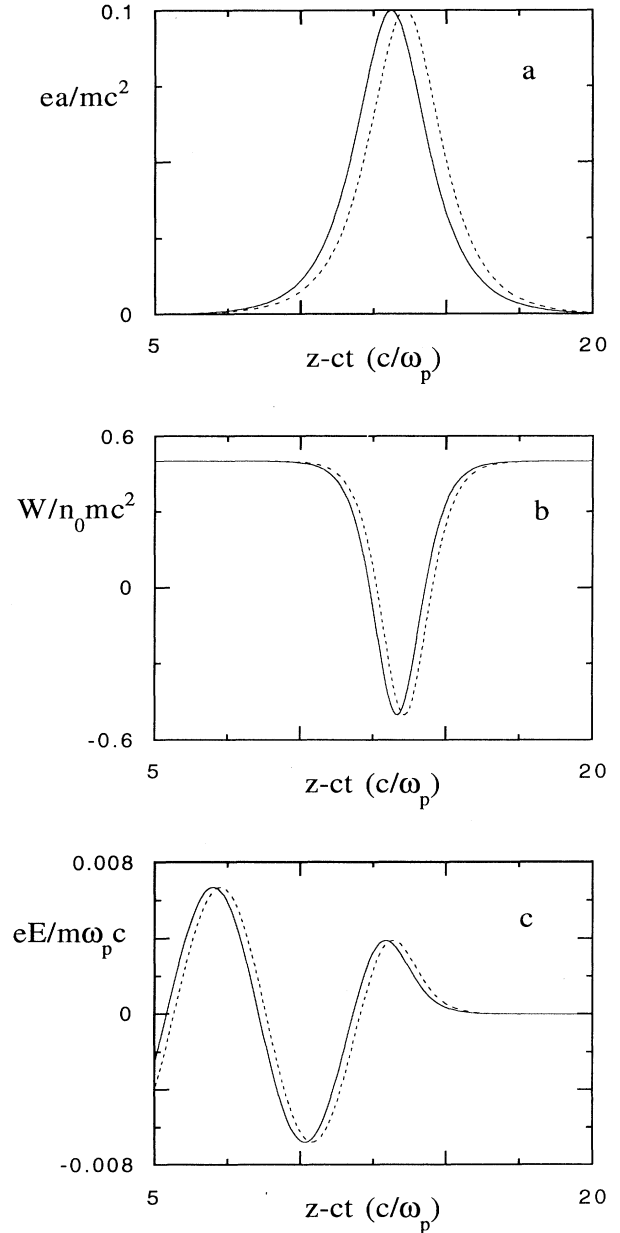


FIG. 1. (a) Initial sech laser pulse (—), and that laser pulse at  $\omega_p t = 50$  (---) for the initial conditions of  $W_0 = 0.5n_0 mc^2$  (theoretical value for  $\beta = 1.01$ ). (b), (c) Energy density of the active medium (wake field) at  $\omega_p t = 25$  (—) and  $\omega_p t = 50$  (---). Initial conditions as used in Table I.

of the laser pulse was about  $4c/\omega_p$ , approximately equal to that predicted by Eq. (17).

Figure 1(a) shows the results for  $v_{\text{env}}=1.01c$  ( $W_0=0.5n_0mc^2$ ) with all parameters as above for the initial laser pulse and the pulse at  $\omega_p t=50$ . Note the advancement of the envelope in this moving frame, indicating an envelope velocity greater than  $c$ . Figure 1(b) shows the energy density of the active medium at  $\omega_p t=25$  and  $\omega_p t=50$ , and Fig. 1(c) shows the wake field at time  $\omega_p t=25$  and  $\omega_p t=50$ . Note the accelerated phase velocity observed by an advancement of the wake field in this moving frame, also indicating that its phase velocity is greater than  $c$  ( $v_{\text{phase}}=1.01c$ ). Also note the complete return of the energy density to its value at infinity, as is necessary for a stationary structure.

The second condition is that of a finite laser pulse [3]; that is, a sech laser pulse with a sharp leading edge. Simulations show the velocity of the peak portion of the laser pulse travels with the design velocity ( $v_{\text{des}}$ ) if the peak is far from the sharp edge. This edge travels with the normal group velocity as the peak portion travels up towards the sharp edge. As the peak approaches the leading edge, its velocity is reduced from the design velocity and approaches the normal group velocity. When the peak has caught up with the leading edge, its velocity is now equal to that of the edge, the normal group velocity. Figure 2 shows a sech laser pulse with a finite leading edge ( $a=0$  at  $\lambda \approx 18c/\omega_p$ ) and compares its evolution to that of the same sech with an infinite leading edge. The simulation shows that the peak of the finite pulse travels up to the leading edge and the velocity of the sharp edge is equal to the normal group velocity, which in this case is  $v_g=c$ , since there is no plasma and  $\eta=1.0$ . Initially the peak starts at the design velocity ( $v_{\text{env}}=1.01c$ ), but slows down as it approaches the leading edge of the laser pulse. Upon reaching the leading edge the peak travels with the normal group velocity and is amplified. The same figure also shows where the same laser pulse would

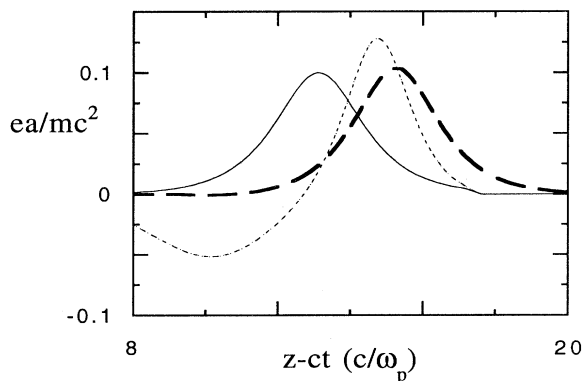


FIG. 2. Initial finite sech pulse (—) with a sharp leading edge (edge at  $\approx 18c/\omega_p$ ). Also shown, that finite pulse at time  $\omega_p t=220$  (---) and the position of the same initial sech pulse if it had been infinite also at time  $\omega_p t=220$  (-·-·-). Note that the leading edge of the finite pulse travels at  $v_g=c$ , since there is no plasma and  $\eta=1.0$ . The initial energy density of the active medium is  $W_0=0.5n_0mc^2$ ,  $\omega_0/\omega_p=100$ ,  $\eta=1$ , and  $\kappa=0.19 e/mc$ .

be if it had an infinite leading edge. Note that the infinite pulse has traveled past the finite pulse with the envelope velocity  $v_{\text{env}}=1.01c$ .

#### IV. APPLICATIONS TO WAKE-FIELD ACCELERATION

One of the major problems with the laser wake-field acceleration (LWFA) is that the laser pulse and therefore the phase velocity of the wake field does not travel at the speed of light  $c$  in a nonactive plasma. This causes the ultrarelativistic particles to dephase with the accelerating field as the accelerated particles outrun the accelerating field [8]. For application of wake-field acceleration we show that the laser envelope and wake field are “accelerated” to the speed of light  $c$  in an active-plasma medium ( $v_{\text{env}}=c$ ) in accordance with Eq. (19) from its original velocity of  $v_g < c$ . Figures 3(a) and 3(b) show an initial sech laser pulse and the same pulse at time  $\omega_p t=50$  with  $\omega_0/\omega_p=10$  and no active medium. The simulation shows that the pulse traveled with the normal group velocity of  $v_{\text{normal}}=0.995c$ , as expected (this corresponds to a velocity of  $v_{\text{normal}}=-0.005$  in the frame moving with the speed of light). The laser pulse is retarded in this moving frame, as is the wake field. This reduction in pulse velocity from the speed of light causes a reduction in the phase velocity of the wake field ( $v_{\text{phase}}=0.995c$ ), thereby causing phase slippage between the ultrarelativistic particles and the accelerating field. In Figs. 3(c) and 3(d) we show the same laser pulse as above, except in an active-plasma medium. The plasma parameters are as above, and the energy density of the active medium is given according to Eq. (19) ( $W_0=0.0025n_0mc^2$ ) to accelerate the envelope velocity to the speed of light. The simulation recorded a velocity of  $v_g=1.000c$ . Also note that the phase velocity of the accelerating field is now the speed of light and there is no slippage between the fields in this moving frame. This will eliminate the problem of phase slippage between the accelerating field and the particles to be accelerated.

The last parameter we looked at is the evolution of a Gaussian laser pulse. Since it is not the stationary form, the pulse shape will evolve in time. The important parameters here are the way the form of the pulse evolves and the time scale for that evolution. The importance of the way the pulse shape changes depends upon the application. Figures 4(a) and 4(b) show a Gaussian laser pulse in a nonactive medium shown with  $\omega_0/\omega_p=10$ , having a group velocity in the simulation of  $v_g=0.995c$ . Again we see the phase velocity of the wake field less than  $c$  ( $v_p=0.995c$ ). Since a Gaussian shaped laser pulse is not the shape for a stationary structure, we want to study the stability of the pulse in an active medium. In Figs. 4(c) and 4(d) we see the evolution of a Gaussian laser pulse in an active medium with an energy density of  $W_0=0.0025n_0mc^2$ , the same value as in Figs. 3(c) and 3(d) to accelerate the sech laser pulse to the speed of light  $c$ . Note that the peak portion of the laser pulse travels at the design velocity  $c$ . However, the front and tail of the laser pulse do not travel at  $c$ . Also the amplitude of the peak increases to evolve towards the stationary form of a sech. For application to acceleration it is the peak por-

tion of the laser pulse that induces the wake and therefore controls the wake-field amplitude and phase velocity. This can be seen, as there is almost no dephasing in the moving frame between the wake field at the end and at the earlier time, when compared to the nonaccelerated case, which has considerable dephasing. Since the Gaussian pulse is not the stationary structure, a higher energy density is necessary than that given by Eq. (19). We see that the velocity of the peak portion of the Gaussian laser pulse is stable for a long period of time and may be used to accelerate the phase velocity of the wake field, overcoming the problem of phase slippage between particles and the accelerating field.

### V. ACTIVE-PLASMA MEDIUM

A number of conditions must be met for a material to be used as the active medium-plasma source. The basic concept we suggest here is to use the outer shell electrons of atoms to form a plasma and inner shell electron resonant transitions as the active source, as in the basic scheme used in x-ray lasers [14]. The inner shell transition energy of the active material must match that of the laser frequency (resonance). For applications using high intensity laser pulses, the problems of the competing pro-

cesses of ionization, power broadening of the resonance, and locating resonances in an intense electric field must be addressed.

For a resonance to be sharp enough to allow the lasing transitions, the maximum electric field of the laser must be small when compared to the atomic ion field. Using the simple Bohr model of the atom ionized to a charge of  $Z-1$ , the electric field at the outermost electron (resonance electron) is approximately

$$E_{\text{atom}} = \frac{Z^3 e}{n^{*4} a_0^2}, \quad (22)$$

where  $n^*$  is the effective principle quantum number and  $a_0$  is the Bohr radius. In this model we assume for simplicity a single outer electron with a core of the remaining un-ionized electrons and the nucleus. If this outer electron is in the closest shell to the core, then  $n^* \approx 1$ . For the atomic electric field to be 10 (100) times that of the peak laser field, the critical intensity is

$$\left. \begin{array}{l} I_{c10} \\ I_{c100} \end{array} \right\} = \left\{ \begin{array}{l} 1/10 \\ 1/100 \end{array} \right\} \times \frac{Z^6 e^2 c}{n^{*8} a_0^2}. \quad (23)$$

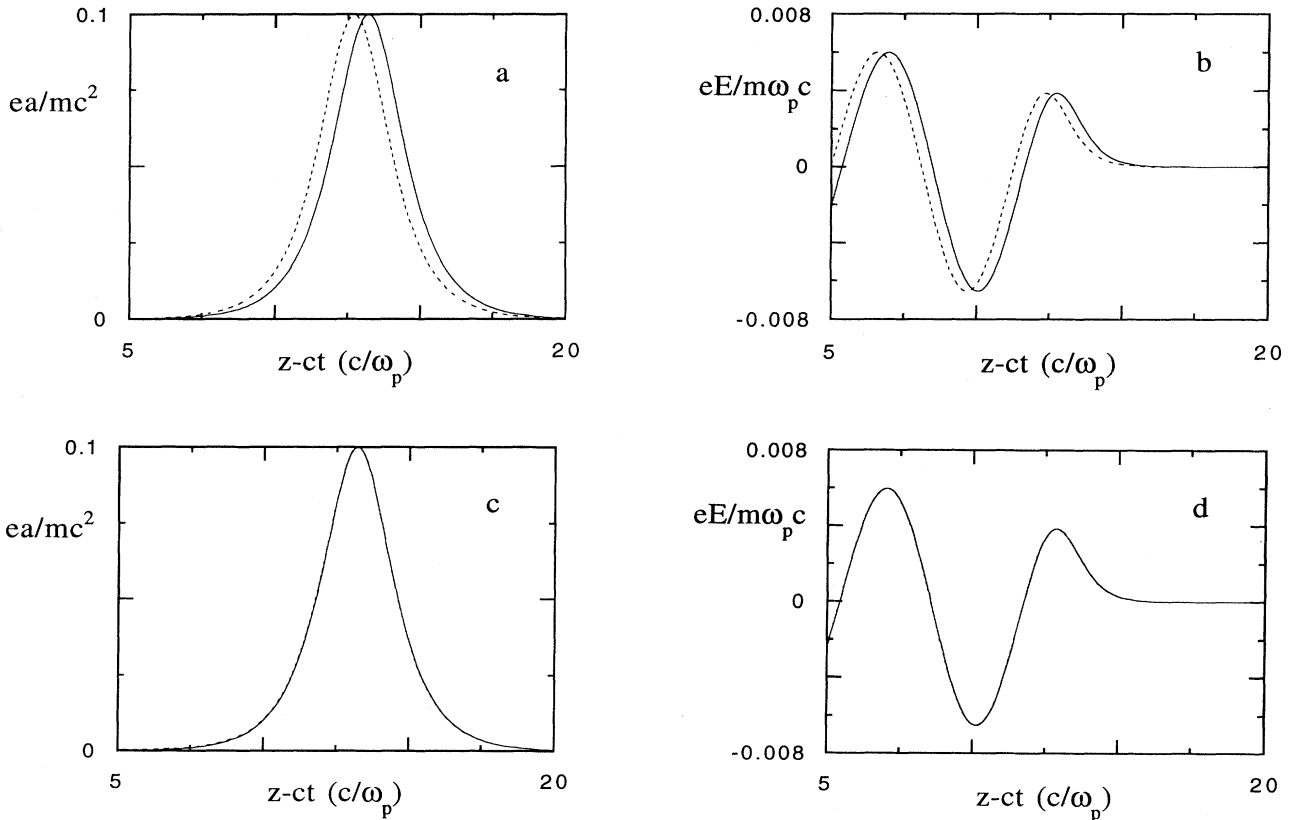


FIG. 3. (a) Initial laser pulse (—) and at  $\omega_p t = 50$  (---) for a sech laser pulse with  $\omega_0/\omega_p = 10$  showing reduction from the vacuum velocity due to the plasma with no active medium. (b) Reduction in the phase velocity of the wake field at time  $\omega_p t = 25$  (solid line) and final time  $\omega_p t = 50$  (---). (c) Initial laser pulse (—) and that pulse at  $\omega_p t = 50$  (---) as above, however, now in an active-plasma medium with energy density  $W_0 = 0.0025 n_0 m c^2$ , the amount required according to Eq. (19) to accelerate the envelope to  $v_g = c$ . (d) The wake field at time  $\omega_p t = 25$  (—) and at time  $\omega_p t = 50$  (---) showing a phase velocity of  $c$ . Note that the two wave forms are coincident for (c) and (d).

Considering a laser pulse with a peak intensity of  $10^{18}$  W/cm<sup>2</sup>, we find that the atom must be ionized to 4 (8) for the atomic field to be 10 (100) times that of the peak laser field according to Eq. (23), with  $n^*=1$ . This seems realizable.

In addition ionization takes place when the Coulomb barrier is suppressed by the laser field at a critical strength, and the atomic electron can freely escape. According to the Coulomb barrier suppression (CBS) theory [15] this corresponds to a laser intensity threshold of

$$I_{th} \text{ (W/cm}^2\text{)} = 4.00 \times 10^9 \frac{[E^4 \text{ (eV)}]}{Z^2}, \quad (24)$$

where  $E = Z^2/2n^{*2}$  or as above when the atomic and laser fields are about equal. Numerical simulations based on the CBS model (see Fig. 5) show that for a 100 fs laser pulse at an intensity of  $10^{18}$  W/cm<sup>2</sup> the degree to which ionization will take place from this pulse is 8 (Ne) to 19 (Xe). Atoms ionized to above these levels will have their outermost surviving electrons (un-ionized), which may be used for the laser resonance.

Power broadening of the energy levels due to the Stark shift [16] by the laser electric field is proportional to the laser power  $E^2$ . This gives the Stark shift  $\Delta\mathcal{E}_{\text{Stark}}$  of the level with energy  $\mathcal{E}$

$$\Delta\mathcal{E}_{\text{Stark}} \approx \mathcal{E} \left[ \frac{E_{\text{atom}}}{E} \right]^2, \quad (25)$$

where  $E_{\text{atom}}$  is the electric field of the atom or ion at the radius of the electron under consideration. The requirement that  $\Delta\mathcal{E}_{\text{Stark}} \leq \epsilon_{\text{res}}/10$  gives another condition for the laser intensity  $I_{\text{Stark}}$ ,

$$I_{\text{Stark}} \leq \frac{1}{10} \frac{\Delta\mathcal{E}_{\text{res}} E_{\text{atom}}^2}{\mathcal{E} 4\pi}. \quad (26)$$

Given  $E_{\text{atom}}/E \approx 10$  (100), the shift in the energy will be approximately  $10^{-2}$  ( $10^{-4}$ ), and for an energy level of approximately 1000 eV the corresponding energy shift is approximately 10 (0.1) eV. With the requirement that the shift be no more than  $\frac{1}{10}$  the energy of the laser for a 1–10 eV photon resonance, the  $E_{\text{atom}}/E$  has to be approximately 100, giving an ionization depth of about 10 electrons.

There are other conditions which, although not necessary, would help to optimize the application for a wake-field accelerator. Since the interaction of the laser with the plasma (ponderomotive force) is proportional to  $(\omega_0/\omega_p)^2$ , for the strongest wake field we prefer  $\omega_0/\omega_p \geq 1$ . The optimal pulse length for wake-field production is  $l_{\text{pulse}} \approx c/\omega_p$ , although our latest work [17] shows that  $l_{\text{pulse}}$  can be substantially longer than this in a

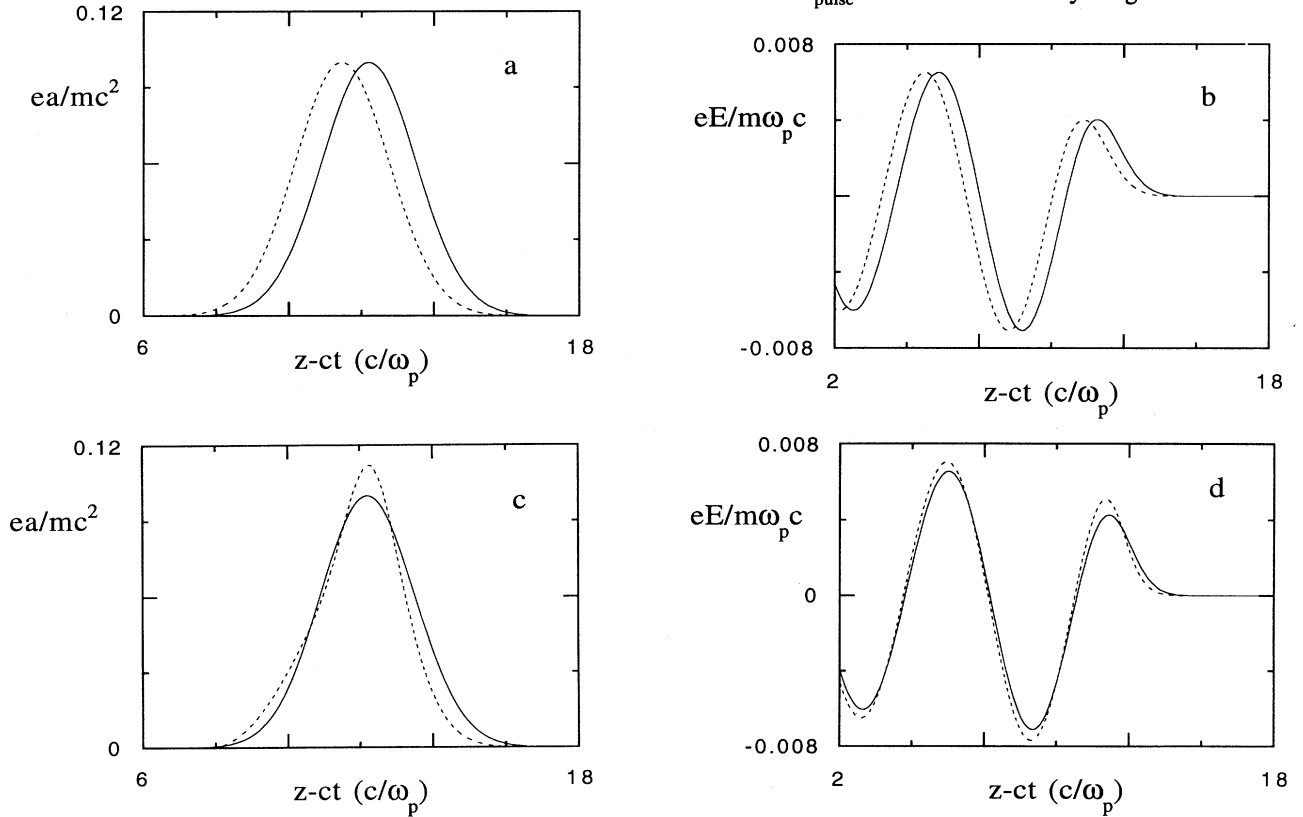


FIG. 4. (a) Initial Gaussian laser pulse (solid line) and that pulse at  $\omega_p t = 50$  (---) with  $\omega_0/\omega_p = 10$  showing reduction from the vacuum velocity due to the plasma with no active medium. (b) Reduction in the phase velocity of the wake field with a time of  $\omega_p t = 25$  (solid line) and final time  $\omega_p t = 50$  (---). (c) Initial laser pulse (—) and that pulse at  $\Omega_p t = 50$  (---) as above, but in an active medium [as in 3(c) and 3(d)]. (d) The wake field at time  $\omega_p t = 25$  (—) and final time  $\omega_p t = 50$  (---) showing a phase velocity of  $c$ .

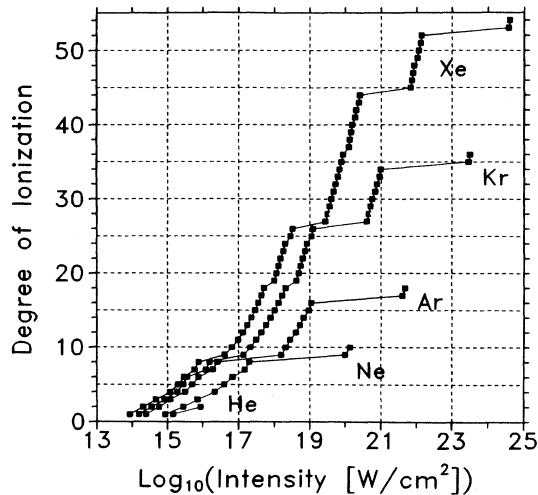


FIG. 5. Degree of ionization (electrons ionized) for noble gases according to CBS theory for a particular intensity ( $\text{W}/\text{cm}^2$ ).

sufficiently strong laser intensity regime. This translates to a present day short pulse of approximately 100 fs. Near future lengths of 10 fs make the preferred transition and therefore laser photon energy in the visible to ultraviolet region. For these low energy resonance transitions in highly ionized atoms we want transitions corresponding only to  $\Delta l$  or  $\Delta m$  (as in the argon ion laser, which uses the  $4p$  to  $4s$  line) but not those with change in principle quantum number  $n$ . Transitions corresponding to  $\Delta n$  are in the x-ray regime and require future development of x-ray lasers to be employed for this application.

The synthesis of the material for use (ionization, activation, etc.) might involve several stages of preparation. Initial formation of the plasma and the ion species could be accomplished by a number of methods, such as a long laser pulse or strong static field. Next the material could be activated (if not already accomplished by the first step) through another laser pulse [16] at the resonance frequency, by electric charge [17], by cascading of electrons down to fill the inversion level or some physical mechanism such as collisions.

A hypothetical example of a possible candidate for a material is argon. Coulomb barrier suppression simulation show that at  $10^{18} \text{ W}/\text{cm}^2$  argon will not be ionized past its outer shell 8 electrons for a 100 fs laser pulse. A long ionization prepulse could ionize all the outer shell electrons (8) and one of the next inner shell electrons. We would then look for possible inversions in the  $2p$ - $2s$  transition. Alternatively we could ionize the outer shell of argon and activate one of the lower level electrons to the now empty 3 shell, looking for an inversion in this shell. This would take a  $\Delta n$  transition to activate and use a  $\Delta l$  in the third shell as the active transition. The  $\Delta n$  transition may possibly be accomplished through collisions, multiphoton absorption of a laser pulse, or a high energy photon laser (x ray). These laser pulses (for activation) would not necessarily need to be short or very high power.

Using our simple model, it is possible to construct a

laser pulse in an active medium without pump depletion whose group velocity is adjustable to the speed of light  $c$ . In the limit that the plasma density goes to zero ( $\phi \rightarrow 0$ ), this stationary structure has the form  $a^2 \propto \text{sech}^2$  [4]. Since the energy transferred to the plasma is a small portion of the energy of the pulse, we expect the stationary structure with a plasma to be similar to the  $\text{sech}^2$ . Simulation results show this holds for our example [1]. Since the plasma does not couple directly to the active medium, the energy to form the wake field originates in the active medium, then flows to the laser pulse, and finally to the wake field.

An alternate approach would be to use an alternating series of accelerating regions (plasma), each followed by an active medium region (nonplasma) used to shape the laser pulse [8]. The laser intensity in the active region could be reduced using optics to expand the laser pulse and then focus down after the pulse has left the active medium region. This would eliminate the problems associated with the high intensity laser pulse. In this case the phase velocity of the wake field would be that of a normal plasma (less than  $c$ ), and therefore the accelerating length would have to be less than the dephasing length. This would still be able to overcome the problems of pump depletion and focusing.

Finally, we discuss the application of an active medium in conjunction with a so far unexplained phenomena observed by Braun *et al.* [18]. In their experiment they observed focusing of  $10^{14} \text{ W}/\text{cm}^2$  laser pulse in air over a distance of 20 m. The focused pulse has a sharp region of high intensity riding on a large but low-intensity pedestal. The intensity in the pedestal is a factor of about 1000 less than the peak. If the pedestal is removed from the laser pulse, however, it is no longer focused and optically guided [19]. This may suggest that the energy in the pedestal is being fed to the main pulse allowing it to keep its structure. It may be possible to use this pedestal as a storage of energy to be supplied to the laser pulse. Since the area of the pedestal is a factor of about 1000 larger than the area of the peaked portion of the laser pulse, storing the energy in the pedestal region would reduce the requirements for the energy density of the active medium by a factor of the ratio of the areas (a factor of 1000 in this case), also allowing the energy storage of the active medium to be in a region of reduced field strength.

## VI. CONCLUSION

In conclusion we find that with an appropriately constructed laser pulse in an active medium we are able to control the envelope velocity to any desired level, including greater than the speed of light  $c$ . We show that it is possible to use this to overcome the dephasing of accelerated particles in a LWFA scheme by increasing the phase velocity of the accelerating field by controlling the envelope velocity of the laser pulse. It is shown that even a nonstationary shape (Gaussian) may be used to accelerate the phase velocity of the wake field. Finally, the present solution of accelerated group velocity and the amplification of the pulse may be applicable to other situations, such as photonics and telecommunications [18]. The present model of the two level atoms as well as other



idealizations is a first step toward understanding the resonant interaction of the laser with the medium to create a superluminous pulse. Other effects such as multilevel atomic states, ionization [19], and more precise identification of appropriate materials have to be ad-

dressed experimentally and by more detailed and perhaps less transparent theories in future investigations.

#### ACKNOWLEDGMENT

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